

FIND ALL IMAGINARY SOLUTIONS TO A POLYNOMIAL EQUATION

$$x^3 - 8 = 0$$

$$x^3 - 2^3 = 0$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$\Rightarrow (x-2)(x^2 + 2x + 4) = 0$$

$$x-2=0 \Rightarrow x=2 \quad \text{Real solution}$$

$$x^2 + 2x + 4 = 0 \Rightarrow x = \frac{-2 \pm \sqrt{4-16}}{2}$$

$$= \frac{-2 \pm \sqrt{0-12}}{2}$$

$$= \frac{-2 \pm \sqrt{2i\sqrt{3}}}{2}$$

$$= -1 \pm i\sqrt{3}$$

$$\left\{ -1 - i\sqrt{3}, -1 + i\sqrt{3} \right\} \text{ imaginary solutions}$$

$$16x^4 - 81 = 0$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$(4x^2)^2 - (9)^2 = 0 \Rightarrow (4x^2 - 9)(4x^2 + 9) = 0$$

$$(2x-3)(2x+3)(4x^2+9) = 0$$

$$2x-3=0 \Rightarrow x = \frac{3}{2} \quad \left. \vphantom{2x-3=0} \right\} \text{ real solutions}$$

$$2x+3=0 \Rightarrow x = -\frac{3}{2}$$

$$4x^2+9=0 \Rightarrow \sqrt{x^2} = \sqrt{-\frac{9}{4}}$$

$$x = \pm \frac{3}{2}i$$

$$\left\{ -\frac{3}{2}i, \frac{3}{2}i \right\} \text{ imaginary solutions}$$

$$x^4 + 5x^2 - 36 = 0$$

$$(x^2 - 4)(x^2 + 9) = 0$$

$$x^2 - 4 = 0 \Rightarrow \sqrt{x^2} = \sqrt{4}$$

$$x = \pm 2 \text{ real solutions}$$

$$x^2 + 9 = 0 \Rightarrow \sqrt{x^2} = \sqrt{-9}$$

$$x = \pm 3i \quad \{-3i, 3i\} \text{ imaginary solutions}$$

$$4x^2 + 11x + 8 = 0$$

$$x = \frac{-11 \pm \sqrt{121 - 4(4)(8)}}{8} = \frac{-11 \pm \sqrt{121 - 128}}{8}$$
$$= \frac{-11 \pm \sqrt{-7}}{8} \begin{cases} -\frac{11}{8} + \frac{\sqrt{7}}{8}i \\ -\frac{11}{8} - \frac{\sqrt{7}}{8}i \end{cases}$$

$$\left\{ -\frac{11}{8} - \frac{\sqrt{7}}{8}i ; -\frac{11}{8} + \frac{\sqrt{7}}{8}i \right\} \text{ imaginary solutions}$$