

SOLVE THE LOGARITHMIC EQUATIONS

$$\log_a (x-1) - \log_a (x+6) = \log_a (x-2) - \log_a (x+3)$$

$$x-1 > 0$$

$$x+6 > 0$$

$$x-2 > 0$$

$$x+3 > 0$$

$$x > 1$$

$$x > -6$$

$$x > 2$$

$$x > -3$$

$$(1, \infty)$$

$\cap$

$$(-6, \infty)$$

$\cap$

$$(2, \infty)$$

$\cap$

$$(-3, \infty)$$

$$= (2, \infty)$$

$$\log_a \frac{x-1}{x+6} = \log_a \frac{x-2}{x+3}$$

$$\cancel{(x+3)} \cancel{(x+6)} \frac{x-1}{\cancel{x+6}} = \frac{x-2}{x+3} \quad \cancel{(x+3)} \cancel{(x+6)}$$

$$(x+3)(x-1) = (x-2)(x+6)$$

$$x^2 + 2x - 3 = x^2 + 4x - 12$$

$$-x^2 - 4x + 12 \quad -x^2 - 4x + 12$$

$$-2x + 9 = 0$$

$$-2x = -9$$

$$x = \frac{9}{2} \in (2, \infty) \checkmark$$

$$\log_2 (x+7) + \log_2 (x+8) = 1$$

$$x > -7$$

$$x > -8$$

$$(-7, \infty)$$

$$\cap$$

$$(-8, \infty)$$

$=$

$$(-7, \infty)$$

$$\log_2 (x+7)(x+8) = 2^1$$

$$(x+7)(x+8) = 2^1$$

$$x^2 + 15x + 56 = 2$$

$$x^2 + 15x + 54 = 0$$

$$(x+9)(x+6) = 0$$

$$x = -9 \quad x = -6$$

$$x = -9 \notin (-7, \infty)$$

EXTRANEUS

$$x = -6 \in (-7, \infty)$$

SOLVE THE EXPONENTIAL EQUATIONS

$$5 \cdot 2^{3x} = 8$$

$$2^{3x} = \frac{8}{5}$$

$$\ln 2^{3x} = \ln \frac{8}{5}$$

$$3x \cdot \frac{\ln 2}{3 \ln 2} = \ln \frac{8}{5}$$

$$x = \frac{\ln \frac{8}{5}}{3 \ln 2}$$

$$\pi^{1-x} = e^x$$

$$\ln \pi^{(1-x)} = \ln e^x$$

$$(1-x) \ln \pi = x$$

$$\ln \pi - x \ln \pi = x$$

$$\ln \pi = x + x \ln \pi$$

$$\frac{\ln \pi}{1 + \ln \pi} = \frac{x(1 + \ln \pi)}{1 + \ln \pi}$$

$$x = \frac{\ln \pi}{1 + \ln \pi}$$

$$2 \cdot 49^x + 11 \cdot 7^x + 5 = 0$$

$$2(7^2)^x + 11 \cdot 7^x + 5 = 0$$

$$2 \cdot (7^x)^2 + 11(7^x) + 5 = 0 \Rightarrow 7^x = u$$

$$2u^2 + 11u + 5 = 0$$

$$7^x = -5$$

$$7^x = -\frac{1}{2}$$

$$2u^2 + 10u + u + 5 = 0$$

$$\ln 7^x = \ln(-5)$$

$$\ln 7^x = \ln\left(-\frac{1}{2}\right)$$

$$2u(u+5) + 1(u+5) = 0$$

$$(u+5)(2u+1) = 0$$

$$u = -5 \quad u = -\frac{1}{2}$$

$\Rightarrow 2 \cdot 49^x + 11 \cdot 7^x + 5 = 0$  has no real solutions