

a) FIND THE INVERSE OF THE FUNCTION

$$f(x) = \frac{2x-3}{x-4}$$

$$y = \frac{2x-3}{x-4} \Rightarrow x = \frac{2y-3}{y-4} \quad | \cdot (y-4)$$

$$\cancel{x}y - 4\cancel{x} = \frac{2y-3}{y-4}$$

$$xy - 2y = 4x - 3$$

$$y(x-2) = 4x-3$$

$$y = \frac{4x-3}{x-2}$$

$$\Rightarrow f^{-1}(x) = \frac{4x-3}{x-2}$$

$$(f \circ f^{-1})(x) = f\left(\frac{4x-3}{x-2}\right) = \frac{2 \cdot \frac{4x-3}{x-2} - 3}{\frac{4x-3}{x-2} - 4} \cdot \frac{x-2}{x-2} = \frac{8x-6-3x-6}{4x-3-4x+8} = \frac{5x-12}{x-2}$$

$$(f^{-1} \circ f)(x) = \frac{4 \cdot \frac{2x-3}{x-4} - 3}{\frac{2x-3}{x-4} - 2} \cdot \frac{x-4}{x-4} = \frac{8x-12-3x+12}{2x-3-2x+8} = \frac{5x}{5} = x$$

b) FIND THE DOMAIN AND THE RANGE OF $f(x)$.

Domain of $f(x)$ $x-4 \neq 0 \quad x \neq 4$
 $(-\infty, 4) \cup (4, \infty)$

Range of $f(x) = \text{Domain } f^{-1}(x) \Rightarrow$

$x-2 \neq 0$
 $x \neq 2$ $(-\infty, 2) \cup (2, \infty)$ range of $f(x)$