

FIND ALL REAL SOLUTIONS FOR THE FOLLOWING EQUATIONS:

$$\sqrt[3]{x-1} + 1 = 0$$

$$\left(\sqrt[3]{x-1}\right)^3 = (-1)^3$$

$$x-1 = -1$$

$$\begin{array}{r} +1 \\ +1 \end{array}$$

$$\boxed{x=0}$$

check: $\sqrt[3]{0-1} + 1 \stackrel{?}{=} 0$

$$\sqrt[3]{-1} + 1 \stackrel{?}{=} 0$$

$$-1 + 1 \stackrel{?}{=} 0$$

$$0 = 0 \checkmark$$

LS RS.

$$\sqrt[4]{2x+2} = 2$$

$$\left(\sqrt[4]{2x+2}\right)^4 = (2)^4$$

$$2x+2 = 16$$

$$2x = 14$$

$$\boxed{x=7}$$

check: $\sqrt[4]{2 \cdot 7 + 2} \stackrel{?}{=} 2$

$$\sqrt[4]{16} \stackrel{?}{=} 2$$

$$2 = 2 \checkmark$$

$$\sqrt{4-x} = x-2$$

$$\left(\sqrt{4-x}\right)^2 = (x-2)^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$4-x = x^2 - 4x + 4$$

$$\begin{array}{r} -4+x \\ +x-4 \end{array}$$

$$0 = x^2 - 3x$$

$$0 = x(x-3)$$

$x \geq 0$ EXTRANEOUS

$$x-3=0 \Rightarrow \boxed{x=3}$$

check: $x=0$ $\sqrt{4-0} \stackrel{?}{=} 0-2$

$$\sqrt{4} \stackrel{?}{=} -2$$

$$2 \neq -2$$

$\boxed{x=3}$ $\sqrt{4-3} \stackrel{?}{=} 3-2$

$$\sqrt{1} = 1$$

$$1 = 1 \checkmark$$

$$\sqrt{2x+5} + \sqrt{2x} = 3$$

$$\sqrt{2x+5} = 3 - \sqrt{2x}$$

$$(\sqrt{2x+5})^2 = (3 - \sqrt{2x})^2$$

$$2x+5 = 9 - 6\sqrt{2x} + 2x$$

$$-4 = -6\sqrt{2x} \quad \left| \cdot \left(-\frac{1}{2}\right)\right.$$

$$(2)^2 = (3 - \sqrt{2x})^2$$

$$4 = 9 - 2x$$

$$4 = 18 - 2x$$

$$x = \frac{4 \div 2}{18 \div 2}$$

$$\boxed{x = \frac{2}{9}}$$

$$\text{check: } \sqrt{2 \cdot \frac{2}{9} + 5} + \sqrt{2 \cdot \frac{2}{9}} \stackrel{?}{=} 3$$

$$\sqrt{\frac{4}{9} + \frac{45}{9}} + \sqrt{\frac{4}{9}} \stackrel{?}{=} 3$$

$$\sqrt{\frac{49}{9}} + \sqrt{\frac{4}{9}} \stackrel{?}{=} 3$$

$$\frac{7}{3} + \frac{2}{3} \stackrel{?}{=} 3$$

$$\frac{9}{3} \stackrel{?}{=} 3$$

$$3 = 3 \checkmark$$

$$\left(\sqrt[3]{-6x-1}\right)^3 = \left(\sqrt[3]{-2x-5}\right)^3$$

$$-6x-1 = -2x-5$$

$$+2x+1 \quad +2x+1$$

$$-4x = -4$$

$$\boxed{x = 1}$$

$$\text{check: } \sqrt[3]{-6 \cdot 1 - 1} = \sqrt[3]{-2 \cdot 1 - 5}$$

$$\sqrt[3]{-6-1} = \sqrt[3]{-2-5}$$

$$\sqrt[3]{-7} = \sqrt[3]{-7} \checkmark$$

FIND THE REAL SOLUTIONS:

$$x^4 + 2x^2 - 3 = 0$$

$$(x^2)^2 + 2(x^2) - 3 = 0$$

$x^2 = u$ substitution

$$u^2 + 2u - 3 = 0$$

$$(u+3)(u-1) = 0$$

$$u+3=0 \Rightarrow u=-3 \Rightarrow \sqrt{x^2} = \sqrt{-3} \quad x = \sqrt{-3} \quad \text{Not a real solution}$$

$$u-1=0 \Rightarrow u=1 \Rightarrow \sqrt{x^2} = \sqrt{1} \quad \boxed{x = \pm 1}$$

$$x^{\frac{2}{3}} - 3x^{\frac{1}{3}} - 10 = 0$$

$$(x^{\frac{1}{3}})^2 - 3(x^{\frac{1}{3}}) - 10 = 0$$

$$x^{\frac{1}{3}} = u$$

$$x^{\frac{1}{3}} = \sqrt[3]{x}$$

$$u^2 - 3u - 10 = 0$$

$$(u+2)(u-5) = 0$$

$$u+2=0 \Rightarrow u = -2$$

$$x^{\frac{1}{3}} = -2 \Rightarrow (\sqrt[3]{x})^3 = (-2)^3 \Rightarrow \boxed{x = -8}$$

$$u-5=0 \Rightarrow u = 5$$

$$x^{\frac{1}{3}} = 5 \Rightarrow (\sqrt[3]{x})^3 = (5)^3 \Rightarrow \boxed{x = 125}$$

FIND THE REAL SOLUTIONS

$$\begin{aligned}x^{\frac{3}{2}} - x^{\frac{1}{2}} &= 0 \\x^{\frac{1}{2}} \left(x^{\frac{3-1}{2}} - x^{\frac{1-1}{2}} \right) &= 0 \\x^{\frac{1}{2}} (x - 1) &= 0 \\x^{\frac{1}{2}} = 0 &\Rightarrow \boxed{x=0} \\x - 1 = 0 &\Rightarrow \boxed{x=1}\end{aligned}$$

$$\begin{aligned}x^4 - 1 &= 0 \\(x^2 - 1)(x^2 + 1) &= 0 \\(x-1)(x+1)(x^2 + 1) &= 0 \\x - 1 = 0 &\Rightarrow \boxed{x=1} \\x + 1 = 0 &\Rightarrow \boxed{x=-1} \\x^2 + 1 = 0 &\Rightarrow \sqrt{x} = \sqrt{-1} \\&\text{Not a real solution}\end{aligned}$$

$$6x^3 + 24 = 0$$

$$6(x^3 + 4) = 0$$

$$x^3 + 4 = 0$$

$$\sqrt[3]{x} = \sqrt[3]{-4}$$

$$\boxed{x = \sqrt[3]{-4}}$$

$$x^3 + 3x^2 - 4x - 12 = 0$$

$$x^2(x+3) - 4(x+3) = 0$$

$$(x+3)(x^2 - 4) = 0$$

$$(x+3)(x-2)(x+2) = 0$$

$$x+3=0 \Rightarrow \boxed{x=-3}$$

$$x-2=0 \Rightarrow \boxed{x=2}$$

$$x+2=0 \Rightarrow \boxed{x=-2}$$