

# General Equations

## SOLVE EQUATIONS ON THE FORM: $\underline{ax + b = c}$

$$ax + b = c$$

$$ax + \cancel{b} - \cancel{b} = c - b \quad \text{Isolate the variable add/subtract the constant in both sides}$$

$$ax = c - b \quad \text{Combine like terms}$$

$$\frac{ax}{a} = \frac{c-b}{a} \quad \text{Divide by the coefficient}$$

$$x = \frac{c-b}{a} \quad \text{The solution}$$

Example: Solve the equation:

a)  $8a + 3 = 10$

$$8a + \cancel{3} - \cancel{3} = 10 - 3 \quad \text{move constant}$$

$$8a = 7$$

$$\frac{8a}{8} = \frac{7}{8} \quad \text{divide by coefficient}$$

$$a = \frac{7}{8}$$

Check

$$8\left(\frac{7}{8}\right) + 3 = 10$$

$$7 + 3 = 10$$

$$10 = 10$$

b)  $\frac{1}{3} - \frac{3}{5}x = -\frac{1}{2}$

$$\frac{1}{3} - \frac{3}{5}x - \frac{1}{3} = -\frac{1}{2} - \frac{1}{3}$$

$$-\frac{3}{5}x = -\frac{5}{6}$$

$$\left(-\frac{5}{3}\right)\left(-\frac{3}{5}x\right) = \left(-\frac{5}{6}\right)\left(-\frac{5}{3}\right)$$

$$x = \frac{25}{18}$$

Check

$$\frac{1}{3} - \frac{3}{5}\left(\frac{25}{18}\right) = -\frac{1}{2}$$

$$\frac{1}{3} - \frac{5}{6} = -\frac{1}{2}$$

$$\frac{2}{6} - \frac{5}{6} = -\frac{1}{2}$$

$$-\frac{3}{6} = -\frac{1}{2}$$

$$-\frac{1}{2} = -\frac{1}{2}$$

## Exercise 1

a) Solve:  $4w + 11 = 7$

$$\frac{4w}{4} = \frac{-4}{4}$$

$$w = -1$$

check

$$4(-1) + 11 \neq 7$$

$$-4 + 11 \neq 7$$

$$7 \neq 7 \quad \checkmark$$

b) Solve:  $4 + 8x = 28$

$$\frac{8x}{8} = \frac{24}{8}$$

$$x = 3$$

$$4 + 8(3) \neq 28$$

$$4 + 24 \neq 28$$

$$28 \neq 28$$

c) Solve:  $-3x + 4 = -17$

$$\frac{-3x}{-3} = \frac{-21}{-3}$$

$$x = 7$$

Check  $-3(7) + 4 = -17$   
 $-21 + 4 = -17$   
 $-17 = -17 \checkmark$

d) Solve:  $-5w + 8 = -2$

$$\frac{-5w}{-5} = \frac{-10}{-5}$$

$$w = 2$$

check,  $-5(2) + 8 = -2$   
 $-10 + 8 = -2$   
 $-2 = -2 \checkmark$

**SOLVE EQUATIONS ON THE FORM:  $ax + b = cx + d$**

$$ax + b = cx + d$$

$$ax + \cancel{b} - \cancel{b} = cx + d - b$$

Add/subtract one constant in both sides of the equation

$$ax = cx + (d - b)$$

Combine like terms

$$ax - cx = \cancel{cx} + (d - b) - \cancel{cx}$$

Add/ subtract one variable in both sides of the equation

$$(a - c)x = (d - b)$$

Combine like terms

$$\frac{(a-c)x}{(a-c)} = \frac{(d-b)}{(a-c)}$$

Divide both sides by the coefficient

$$x = \frac{d-b}{a-c}$$

The solution

**Example:** Solve the equations

a)  $5x - 4 = 3x - 10$

$$5x - 4 + 4 = 3x - 10 + 4$$

$$5x = 3x - 6$$

$$5x - 3x = 3x - 6 - 3x$$

$$2x = -6$$

$$\frac{2x}{2} = \frac{-6}{2}$$

$$x = -3$$

b)  $8x + 3 - 4x = 5 + x$

$$4x + 3 = 5 + x$$

$$4x + 3 - 3 = 5 + x - 3$$

$$4x = 2 + x$$

$$4x - x = 2$$

$$3x = 2$$

$$\frac{3}{3}x = \frac{2}{3}$$

$$x = \frac{2}{3}$$

combine the variable on the left

**Exercise 2**

a) Solve:  $11x + 60 = 5x + 6$ .

$$\begin{array}{r} -5x \quad -5x \\ 6x + 60 = 6 \\ -60 \quad -60 \end{array}$$

$$\frac{6x}{6} = \frac{-54}{6}$$

$$x = -9$$

$$\begin{array}{r} 11(-9) + 60 = 5(-9) + 6 \\ -99 + 60 = -45 + 6 \\ -39 = -39 \checkmark \end{array}$$

b) Solve:  $8x + 53 = x - 3$ .

$$\begin{array}{r} +3 \quad +3 \\ \hline 56 \end{array}$$

$$\begin{array}{r} 8x + 56 = x \\ -8x \quad -8x \\ \hline 56 = -7 \end{array}$$

$$x = -8$$

c) Solve:  $5x - 5 = 11x - 6$ .

$$5x = 11x - 6 + 5$$

$$5x = 11x - 1$$

$$\begin{array}{r} -11x \quad -11x \end{array}$$

$$\begin{array}{r} -6x = -1 \\ -6 \quad -6 \end{array}$$

$$x = \frac{1}{6}$$

d) Solve:  $\frac{2}{3}x - 1 = \frac{1}{3}x + 4$ .

$$\frac{2}{3}x = \frac{1}{3}x + 5$$

$$\frac{2}{3}x - \frac{1}{3}x = 5$$

$$\frac{3}{3} \cdot \frac{1}{3}x = 5 \cdot \frac{3}{1}$$

$$x = 15$$

$$\left(\frac{2}{3} \cdot 15\right) - 1 = \left(\frac{1}{3} \cdot 15\right) + 4$$

$$10 - 1 = 5 + 4$$

$$9 = 9 \checkmark$$

## SOLVE EQUATIONS CONTAINING PARENTHESES

Example: Solve the equation

a) $9x - (2x + 5) = 4(5x + 2)$	Eliminate parentheses using distributive property
$9x - 2x - 5 = 20x + 8$	Combine like terms
$7x - 5 = 20x + 8$	Add/subtract a variable or a constant from both sides of the equation
$7x - 5 - 7x = 20x + 8 - 7x$	Combine like terms
$-5 = 13x + 8$	Add/Subtract the constant or the variable from both sides of the equation
$-5 - 8 = 13x + 8 - 8$	Combine like terms
$-13 = 13x$	Divide/multiply both side of the equation with the coefficient of the variable
$\frac{-13}{13} = \frac{13x}{13}$	Simplify
$-1 = x$	The solution

Check

$$9(-1) - (2(-1) + 5) = 4(5(-1) + 2)$$

$$-9 - (-2 + 5) = 4(-5 + 2)$$

$$-9 - 3 = 4(-3)$$

$$-12 = -12$$

### Exercise 3

a) Solve:  $18x + 6(x - 1) = 22$ .

$$18x + 6x - 6 = 22$$

$$24x - 6 = 22$$

$$+6 \quad +6$$

$$\frac{24x}{24} = \frac{28}{24} \div 4$$

$$x = \frac{7}{6}$$

b) Solve:  $2r - 8 = 3(3r + 4) - 13$

$$2r - 8 = 9r + 12 - 13$$

$$\begin{array}{r} 2r - 8 = 9r - 1 \\ +1 \quad +1 \\ \hline \end{array}$$

$$\begin{array}{r} 2r - 7 = 9r \\ -2r \quad -2r \\ \hline \end{array}$$

$$-7 = 7r$$

$$\boxed{r = -1}$$

c) Solve:  $3z - 2 = -5(2z - 9) - 8$

$$3z - 2 = -10z + 45 - 8$$

$$\begin{array}{r} 3z - 2 = -10z + 37 \\ +10 \quad +10 \end{array}$$

$$\begin{array}{r} 13z - 2 = 37 \\ +2 \quad +2 \end{array}$$

$$\frac{13z}{13} = \frac{39}{13}$$

$$\boxed{z = 3}$$