RATIONAL EXPRESSIONS
INTRODUCTION TO RATIONAL FUNCTIONS.

Def: A rational function is on the form

$f(x) = \frac{P}{Q}$, $Q \neq 0$ where P and Q are polynomial expressions

Ex: $f(x) = \frac{1}{x-2}$, $g(x) = \frac{x+1}{x^2-2x+1}$

$f(x) = \frac{x^3+7}{x-1}$

Examples of nonrational expressions/functions

$\frac{x^2+2x-1}{2} = \frac{1}{2}x^2 + \frac{2}{2}x - \frac{1}{2}$ polynomial with rational coefficients

not a rational expression!

$\frac{x+5}{\sqrt{x-1}}$ $\sqrt{x-1}$ is not a polynomial.

$\Rightarrow$ is not rational expression

EVALUATE RATIONAL EXPRESSIONS.

$$\frac{x^2}{x-3}, \quad x=1, \quad x=0, \quad x=3$$

$x=1$

$$\frac{x^2}{x-3} = \frac{1^2}{1-3} = \frac{1}{-2} = -\frac{1}{2}$$

$x=0$

$$\frac{x^2}{x-3} = \frac{0^2}{0-3} = \frac{0}{-3} = 0$$

$x=3$

$$\frac{x^2}{x-3} = \frac{3^2}{3-3} = \frac{9}{0}$$ undefined. !!!

Domain of rational expressions / functions

$$\frac{5x-1}{x+2}$$

We must avoid the ~~denominator~~ denominator to be zero so we will find all the values that give us zero at denominator by solving the equation $Q(x)=0$

$x+2=0$

$-2 \quad -2$

$x=-2$

$\{x \mid x \neq -2\}$

$$F(x)=\frac{2x^2+5x+2}{x^2+2x-3}$$

Domain

$x^2+2x-3=0$

$(x-1)(x+3)=0$

$x-1=0$ $\quad$ $x+3=0$

$+1 \quad +1$ $\quad$ $-3 \quad -3$

$x=1$ $\quad$ $x=-3$

F -3 | 5 = 2

-1 . 3 |

$\{x \mid x \neq -3, 1\}$

Problem 2

$(x+2)(x-2)$

$x+2=0 \quad x-2=0$

$-2 \ -2 \quad +2 \ +2$

$x=-2 \quad x=2$

$\frac{5-x}{x^2-4}$

$\{x \mid x \neq -2, 2\}$

Problem 3

$\{x \mid x \in$ real numbers $\}$

$x^2+4=0$ nonfactorable.

SIMPLIFY RATIONAL EXPRESSIONS

$$\frac{9}{12} = \frac{3 \cdot \cancel{3}}{4 \cdot \cancel{3}} = \frac{3}{4}$$

Ex $\dfrac{a^2+5a}{4a+20} = \dfrac{a(a+5)}{4(a+5)} = \dfrac{a}{4} \dfrac{\cancel{(a+5)}}{\cancel{(a+5)}} = \dfrac{a}{4}.$

1. Factor completely each polynomial
2. $\dfrac{P \cdot R}{Q \cdot R} = \dfrac{P}{Q} \boxed{\dfrac{R}{R}}$ $= 1$

Ex: $\dfrac{x^2-3x-28}{x^2-4x-32} = \dfrac{(x-7)\cancel{(x+4)}}{(x-8)\cancel{(x+4)}} = \dfrac{x-7}{x-8}$

$x^2-3x-28 = (x-7)(x+4)$

$x^2-4x-32 = (x-8)(x+4)$

$$\frac{a^{2n}+a^{n}-12}{a^{2n}-2a^{n}-3}=\frac{(a^{n}+4)(a^{n}-3)}{(a^{n}+1)(a^{n}-3)}$$

$$=\frac{a^{n}+4}{a^{n}+1}$$

$$a^{2n}=a^{n}\cdot a^{n}$$

$$a^{2}=a\cdot a$$

Problem 4

(A) $\frac{6x^{4}-24x^{3}}{12x^{3}-48x^{2}}=\frac{6x^{3}(x-4)}{12x^{2}(x-4)}=\frac{6x^{3}\div 6}{12x^{2}\div 6}=\boxed{\frac{1x}{2}}$

(B) $\frac{20x-15x^{2}}{15x^{3}-5x^{2}-20x}=\frac{5x-(3x-4)}{5x(x+3)(3x-4)}=\frac{-1}{(x+3)}$

C. $\frac{X^{2n}+X^{n}-12}{X^{2n}-3X^{n}}$ ① $\frac{(X^{n}+4)(X^{n}-3)}{X^{n}(X^{n}-3)}$ ② $=\frac{X^{n}+4}{X^{n}}$