

Common Factors

A. Factors

$$4 \times 3 = 12$$

4, 3 factors 12 product

Others factors of 12 are: 1, 2, 6, 12 \Rightarrow $1 \times 12 = 12$; $2 \times 6 = 12$

All the factors of 12: 1, 2, 3, 4, 6, 12

$$x \times x = x^2$$

x, x factors x^2 product

Other factors of x^2 are: 1, x^2 $x \cdot x = x^2$ $1 \cdot x^2 = x^2$

All the factors of $x^2 = 1, x, x^2$

Exercise 1: Find all the factors of:

1. $72 = 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72, 8, 9$
 $= 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72$

2. $47 = 1, 47$

3. $36 = 1, 2, 3, 4, 6, 9, 12, 18, 36$

B. The Greatest common factors

Example:

1. Greatest common factor between 12 and 42

12: 1, 2, 3, 4, 6, 12

42: 1, 2, 3, 6, 7, 12, 21, 42

GCF (12, 42) = 12 ✓

2. Greatest common factor between x^2 and x^6

$$\left. \begin{array}{l} x^2 = x \cdot x \\ x^6 = x \cdot x \cdot x \cdot x \cdot x \cdot x \end{array} \right\} \text{GCF}(x^2, x^6) = x^2$$

3. Greatest common factors of $15a^4b^2$ and $18a^2b^3c$

$$\left. \begin{array}{l} 15 = 1, 3, 5, 15 \\ 18 = 1, 2, 3, 6, 9, 18 \end{array} \right\} \text{GCF}(15, 18) = 3$$

$$\left. \begin{array}{l} a^4 = a^2 \cdot a^2 \\ a^2 = a^2 \end{array} \right\} \text{GCF}(a^4, a^2) = a^2$$

$$\left. \begin{array}{l} b^2 = b^2 \\ b^3 = b^2 \cdot b \end{array} \right\} \text{GCF}(b^2, b^3) = b^2$$

Greatest common factors of $15a^4b^2$ and $18a^2b^3c \Rightarrow \text{GCF}(15a^4b^2, 18a^2b^3c) = 3a^2b^2$

The common factor of two or more variables (the same base) is the one with the smaller exponent.

Exercise 1:

a) Find the greatest common factor. $48q^2r^2, 12q^5r^5, 6r^3$

$$\cancel{\text{GCF}(48q^2r^2, 12q^5r^5, 6r^3)} = 6r^2$$

b) Find the greatest common factor. $4x^6y, 18x^2y^6$

$$4: 1, 2, 4$$

$$18: 1, 2, 3, 6, 9, 18$$

$$2x^2y$$

C. Factor a polynomial

Example: Factor $10y^2 - 15y^3z$.

$$\begin{aligned} \text{GCF}(10y^2, 15y^3z) &= 5y^2 \\ \frac{10y^2}{5y^2} &= 2; \quad \frac{15y^3z}{5y^2} = 3yz \\ 10y^2 - 15y^3z &= 5y^2(2 - 3yz) \end{aligned}$$

Polynomial Factors

$$\frac{y^3}{y^2} = y^{3-2} = y.$$

STEPS

1. Find the GCF of each term in the polynomial
2. Divide each term by the GCF
3. Write the polynomial as a product between GCF and the quotient of the divisions.

Exercise 2:

1. Factor: $9z + 36 = 9(z + 4)$

$$\begin{array}{l} 9 = 9 \\ 36 = 9 \cdot 4 \end{array} \quad \frac{9z}{9} = z \quad \frac{36}{9} = 4$$

2. Factor: $3 - 12t^2 = 3(1 - 4t^2)$

$$3(1 - 4t^2) = 3 - 12t^2$$

3. Factor: $2y^3 + 4y^2 + 4y = 2y(y^2 + 2y + 2)$

$$2y^3 = (2y) \cdot y^2$$

$$4y^2 = (2y) \cdot 2y$$

$$4y = (2 \cdot y) \cdot 2$$

4. Factor: $2a^3b - 10a^2b^2 + 4ab^3 = 2ab(a^2 - 5ab + 2b^2)$

5. Factor: $4a^2b^2 - 20ab^2 + 8b^2$.

$$4b^2(a^2 - 5a + 2)$$

6. Factor: $-3x + 6$.

$$\begin{array}{r} 3 \overline{) 6} \\ 6 \\ \hline 0 \end{array}$$

$$\frac{-3x}{3} = -x \quad \frac{6}{3} = 2$$

$$-3(-x + 2) = -3(x - 2)$$

7. Factor: $-10xy - 12y = -2y(5x + 6)$

8. Factor: $-2x^3 - 18x^2y - 20$.

$$2(-x^3 - 9x^2y - 10)$$

$$-2(x^3 + 9x^2y + 10)$$

D. Factor by grouping

This apply for polynomial of 4 or 6 terms. The most common ones are 4 term polynomial.

Example: Factor: $a(b-7) + b(b-7)$

$$GCF[a(b-7), b(b-7)] = \underline{(b-7)}$$

$$\frac{a\cancel{(b-7)}}{\cancel{(b-7)}} = a; \quad \frac{b\cancel{(b-7)}}{\cancel{(b-7)}} = b$$

$$\textcircled{a}\underline{(b-7)} + \textcircled{b}\underline{(b-7)} = \underline{(b-7)}(a+b)$$

Exercise 3:

1. Factor: $y(b+5) + 2(b+5) = \underline{(b+5)}(y+2)$

2. Factor: $t(t-4) + y(4-t) = -t(4-t) + y(4-t)$
 $= \underline{(4-t)}(-t+y) = \underline{(4-t)}(y-t)$

3. Factor: $u(9v-8) + 2(8-9v) = u(9v-8) - 2(9v-8)$
 $\underline{(9v-8)}(u-2)$

4. Factor: $2x(x-3) + 5(x-3)$
 $\underline{(x-3)}(2x+5)$

5. Factor $x(y-5) + 3(5-y)$ $x(y-5) - 3(5-y)$
 $\underline{(y-5)}(x+3)$ $\underline{(y-5)}(x+3)$
 $= x(y-5) + 3(y-5)$

Example:

1. Factor: $(8x^2 - 12x) + (6xy - 9y)$

$$\text{GCF}(8x^2, 12x) = 4x; \quad \text{GCF}(6xy, 9y) = 3y$$

$$\frac{8x^2}{4x} = 2x; \quad \frac{12x}{4x} = 3; \quad \frac{6xy}{3y} = 2x; \quad \frac{9y}{3y} = 3$$

$$8x^2 - 12x - 6xy + 9y = 4x(2x - 3) + 3y(2x - 3) = (2x - 3)(4x + 3y)$$

2. Factor: $8x^2 - 12x + 6xy - 9y$

$$\text{GCF}(8x^2, 6xy) = 2x; \quad \text{GCF}(12x, 9y) = 3$$

$$\frac{8x^2}{2x} = 4x; \quad \frac{6xy}{2x} = 3y; \quad \frac{12x}{3} = 4x; \quad \frac{9y}{3} = 3y$$

$$8x^2 - 12x + 6xy - 9y = 2x(4x + 3y) - 3(4x + 3y) = (4x + 3y)(2x - 3)$$

3. Factors: $27b^2 + 6bz - 45b - 10z$

$$\text{GCF}(27b^2, 6bz) = 3b; \quad \text{GCF}(45b, 10z) = 5$$

$$27b^2 + 6bz - 45b - 10z = 3b(9b + 2z) - 5(9b + 2z) = (9b + 2z)(3b - 5)$$

Exercise 4:

$$1. \text{ Factor: } \overbrace{bz+8b}^b - \underbrace{4z-32}_4 = b(z+8) - 4(z+8) = (z+8)(b-4)$$

$$2. \text{ Factor: } \underbrace{15a^2+6az}_{3a(5a+2z)} - \underbrace{35a-14z}_{7(5a-2z)} = (5a+2z)(3a-7)$$
$$3a(5a+2z) - 7(5a-2z)$$

$$3. \text{ Factor: } \overbrace{5yz+3y-15z^2-9z}_{(y-3z)(5z+3)} = 5z(y-3z) + 3(y-3z)$$
$$(y-3z)(5z+3) \checkmark$$

$$4. \text{ Factor: } \underbrace{7xy^2-3y}_{y(7xy-3)} + \overbrace{14xy-6}^2 = y(7xy-3) + 2(7xy-3) = (7xy-3)(y+2)$$