

## Division of Monomials and Scientific Notation

### A. Division of monomials or Integer Exponents

Example: Simplify

$$\frac{x^3 y}{x^2 z^2} = \frac{x^3 y}{x^2 z^2} = \frac{\cancel{x} \cdot \cancel{x} \cdot x y}{\cancel{x} \cdot \cancel{x} z^2} \text{ or } x^{3-2} \frac{y}{z^2} = \frac{xy}{z^2}$$

$$\frac{8x^2}{4x^5} = \frac{8 \cdot \cancel{x} \cdot \cancel{x}}{4 \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x} = \frac{2 \cdot 1}{1 \cdot x \cdot x} = \frac{2}{x^2}$$

$$\frac{x^7}{x^5} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = x^2$$

$$\frac{x^7}{x^5} = x^{7-5} = x^2$$

### Rule for dividing exponential expressions

If  $m$  and  $n$  are two integers and  $x \neq 0$ , then

$$\frac{x^m}{x^n} = x^{m-n}$$

Example: Simplify

$$\frac{r^7 s^8}{r^6 s} = \frac{r^7}{r^6} \cdot \frac{s^8}{s} = r^{7-6} s^{8-1} = r s^7$$

$$\frac{12z^7}{4z^3} = \frac{12}{4} \frac{z^7}{z^3} = 3 z^{7-3} = 3 z^4$$

$$\frac{x^5}{x^8} = \frac{\overbrace{x \cdot x \cdot x \cdot x \cdot x}^5}{\underbrace{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}_8} = \frac{1}{x \cdot x \cdot x} = \frac{1}{x^3}$$

$$\frac{x^5}{x^8} = x^{5-8} = x^{-3} = \frac{1}{x^3}$$

$$\Rightarrow x^{-3} = \frac{1}{x^3}$$

## Negative exponents

If  $n$  is a positive integer and  $x \neq 0$ , then

$$x^{-n} = \frac{1}{x^n}, \text{ and } \frac{1}{x^{-n}} = x^n.$$

$$\frac{1}{x^{-3}} = \frac{1}{\frac{1}{x^3}} = 1 \cdot \frac{x^3}{1} = x^3$$

Example: Simplify

$$\frac{x^2}{x^5} = x^{2-5} = x^{-3} = \frac{1}{x^3}$$

$$\boxed{\frac{1}{x^{-3}} = x^3}$$

$$\frac{(3x^{-1})^{-3}}{x^{-2}} = \frac{3^{-3} x^3}{x^{-2}} = \frac{\frac{1}{3^3} x^3}{\frac{1}{x^2}} = \frac{\frac{1}{27} x^3}{\frac{1}{x^2}} = \frac{x^3}{27} \cdot \frac{x^2}{1} = \frac{x^5}{27}$$

$$\frac{\downarrow \downarrow \downarrow}{3x^{-1}y^4} = \frac{3 \frac{1}{x} y^4}{\uparrow \frac{6^{-1}x^3y^{-2}}{6} x^3 \frac{1}{y^2}} = \frac{\frac{3y^4}{x}}{\frac{x^3}{6y^2}} = \frac{3y^4}{x} \cdot \frac{6y^2}{x^3} = \frac{18y^6}{x^4}$$

$$\overline{(-3x^4y^{-5})(-2x^{-3}y^{-1})} = 6x^{4+(-3)}y^{(-5+(-1))} = 6xy^{-6} = \frac{6x}{y^6}$$

$$\frac{x^5}{x^5} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = 1$$

$$\frac{x^5}{x^5} = x^{5-5} = x^0$$

$x^0 = 1$

$$a^0 = 1$$

$$5^0 = 1$$

### Zero as an exponent

If  $x \neq 0$ , then  $x^0 = 1$ . The expression  $0^0$  is UNDEFINED.

Example: Simplify

$$-(36)^0 = -1$$

$$36^0 = 1$$

$$(-36)^0 = 1$$

$$-(36)^0 = -1$$

$$(x^{-2}y^3)^0 = x^{(-2) \cdot 0} y^{3 \cdot 0} = x^0 \cdot y^0 = 1 \cdot 1 = 1$$

### Exercise 1:

a) Evaluate  $5^{-3}$ .  $\frac{1}{5^3} = \frac{1}{125}$

b) Evaluate  $\frac{1}{2^{-5}}$ .  $\frac{2^5}{1} = \frac{64}{1} = 64$

c) Simplify:  $\frac{a^9 w^3}{a^4 w^9}$ .  $\frac{a^5}{w^6}$   $a^{9-4} w^{3-9} = a^5 w^{-6}$

d) Simplify:  $\frac{2m^3 r^2 y^2}{7m^5 r y^6}$ .  $\frac{2r}{7m^2 y^4}$   $-10+3$

e) Simplify:  $(-7y^3)(y^{-5})^2$ .  $-7y^3 \cdot y^{-10} = -7y^{-7} = -\frac{7}{y^7}$

f) Simplify:  $\frac{5c^{-2}u^{-5}}{16c^9u^9}$ .  $\frac{5}{16c^{11}u^{14}}$

## B. Scientific Notation

Example:

a) Write the number 4,600,000,000,000 in scientific notation.

$$4,600,000,000,000 = 4.6 \times 10^{12}$$

$$4.6 \times 10^{12}$$

b) Write the number 0.000043 in scientific notation.

$$0.000043 = 4.3 \times 10^{-5}$$

$$4.3 \times 10^{-5}$$

left  $\Rightarrow$  positive exponent  
right  $\Rightarrow$  negative exponent

Very large and very small numbers are encountered in the fields of science and engineering.

In **scientific notation**, a number is expressed as a product of two factors, one a number between 1 and 10 and the other a power of 10. To change a number written in decimal notation to scientific notation, write it in the form  $a \times 10^n$ , where  $a$  is a number between 1 and 10 and  $n$  is an integer.

Example:

Write the number 0.000054 in scientific notation.

$$5.4 \times 10^{-5}$$

Write the number in decimal notation.  $2.8 \times 10^3$

$$2.8 \times 10^3 = 2800$$

Write the number  $5.1 \times 10^{-2}$  in decimal notation.

$$5.1 \times 10^{-2} = 0.051$$

$$\underbrace{0.000000 \dots 00ab}_{n \text{ zeros}} = a.b \times 10^{-n} \text{ negative exponent}$$

$$\underbrace{ab000000 \dots 00}_{n \text{ zeros}} = a.b \times 10^n \text{ positive exponent}$$

$$a.b \times 10^{-n} = 0.0000 \dots 0ab$$

$$a.b \times 10^n = ab0000 \dots 000$$

**Exercise 2:**

- i) Write the number  $4.7 \times 10^{12}$  in decimal notation.

$$4,700,000,000,000$$

- ii) Write the mass of Mars, which is approximately 642,000,000,000,000,000,000 kg, in scientific notation.

$$6.42 \times 10^{23}$$

- iii) The diameter of a dust particle is 0.000000022 meters. Write this number in scientific notation.

$$2.2 \times 10^{-8}$$

- iv) Write the mass of Earth, which is approximately 5,980,000,000,000,000,000,000 kg, in scientific notation.

$$5.98 \times 10^{24}$$

- v) The electric charge on an electron is 0.000000000000000000016 coulomb. Write this number in scientific notation.

$$1.6 \times 10^{-19}$$

**Exercise 3:**

1. Multiply:  $(\underline{3.8} \times 10^{-6})(\underline{9.7} \times 10^8) = (3.8)(9.7) \times (10^{-6})(10^8)$   
 $= 36.86 \times 10^2 = 3.686 \times 10^3$  or 3686

2. Divide:  $\frac{2.356 \times 10^9}{6.2 \times 10^{-4}} = \frac{2.356}{6.2} \times \frac{10^9}{10^{-4}}$   
 $= 0.38 \times 10^{9-(-4)} = 0.38 \times 10^{13}$   
 $= 3.8 \times 10^{12}$

3. Multiply:  $(3.0 \times 10^5)(1.1 \times 10^{-8})$

$$(3.0)(1.1) \times (10^5)(10^{-8})$$
$$3.30 \times 10^{-3}$$

4. Divide:  $\frac{7.2 \times 10^{13}}{2.4 \times 10^{-3}} = 3 \times 10^{16}$

Ex 1/366.

$$\text{Add } (\underline{3x^2} - \underline{2x} - \underline{6}) + (\underline{-x^2} - \underline{3x} + \underline{4})$$

$$\underline{2x^2 - 5x - 2}$$

Ex 2/366.

$$\text{Multiply } (5x^2 y z^4)(2x y^3 z^{-1})(7x^{-2} y^{-2} z^3)$$

$$70x y^2 z^6$$

Ex: 5.

$$\text{Simplify } \frac{3x^4 y z^{-1}}{-12x y^3 z^2} = -\frac{1}{4} \frac{x^3}{y^2 z^3}$$

$$= \frac{3^{1-2}}{-12^{1-1}} \frac{x^4}{x^{4-1}} \frac{y^1}{y^3} \frac{z^{-1}}{z^{2-1}} \quad y^{1-3} = y^{-2}$$

$$\frac{1}{z^2 \cdot z}$$

$$\text{Ex 6 } \frac{(2a^4 b^{-3} c^2)^3}{(2a^3 b^2 c^{-1})^4}$$

$$\frac{8a^{12} b^{-9} c^6}{16a^{12} b^8 c^{-4}} \div 4 = \frac{2}{4} \div 2 = \left(\frac{1}{2}\right)$$

$$a^{12-12} = a^0 = 1$$

$$b^{-9-8} = b^{-17}$$

$$c^{6-(-4)} = c^{10}$$

$$\frac{16^{-11} c^{10}}{2^4}$$

$$\frac{1c^{10}}{2b^{17}} = \frac{c^{10}}{2b^{17}}$$

Ex 3/367

$$\frac{(2a^{-4}b^2)^3}{4a^{-2}b^{-1}} = \frac{8a^{-12}b^6}{4a^{-2}b^{-1}} = 2a^{-10}b^7$$

$$a^{-12 - (-2)} = a^{-10}$$

$$b^{6 - (-1)} = b^7$$

$$= \frac{2b^7}{a^{10}}$$

$$\frac{8a^{-12}b^6}{4a^{-2}b^{-1}} = \frac{8b^6b^1}{4a^{12}a^{-2}} = \frac{2b^7}{a^{10}}$$