

$$d) (g \circ f)(0) = g(f(0)) = g(-3) = \frac{1}{-3+1} = \frac{1}{-2} = -\frac{1}{2}$$

$$f(0) = 4 \cdot 0^2 - 3 = -3$$

$$(g \circ f)(0) = (g \circ f)(x) = \frac{1}{4x^2 - 2} \text{ evaluate for } 0 = \frac{1}{4 \cdot 0^2 - 2} = -\frac{1}{2}$$

Domain of Composite Functions

Domain of $(f \circ g)(x)$ is all the x values

from the domain of $g(x)$ - such that they are in the domain of $f(x)$ as well

$$(f \circ g)(x) = f(g(x))$$

$$\text{Ex: } f(x) = \frac{x}{x-1} \quad g(x) = -\frac{4}{x}$$

$$a) (f \circ g)(x) = f(g(x)) = \frac{-\frac{4}{x}}{-\frac{4}{x} - 1 \cdot \frac{x}{x}} = \frac{-\frac{4}{x}}{\frac{-4-x}{x}} = -\frac{4}{x} \cdot \left(\frac{x}{-4-x} \right) \\ = \frac{-4}{-4-x} = \frac{-4}{-(4+x)} = \frac{4}{4+x}$$

$$\left. \begin{array}{l} \text{Domain } g(x) = \mathbb{R} - \{0\} \\ \text{Domain } \frac{4}{4+x} = \mathbb{R} - \{-4\} \end{array} \right\} \text{Domain } (f \circ g)(x) = \mathbb{R} - \{-4, 0\}$$

$$b) (g \circ f)(x) = g\left(\frac{x}{x-1}\right) = -\frac{4}{\frac{x}{x-1}} = -4 \cdot \frac{x-1}{x} = \frac{-4x+4}{x}$$

$$\left. \begin{array}{l} \text{Domain of } f(x) = \mathbb{R} - \{1\} \\ \text{Domain of } \frac{-4x+4}{x} = \mathbb{R} - \{0\} \end{array} \right\} \text{Domain } (g \circ f)(x) = \mathbb{R} - \{0, 1\}$$

$$c) (f \circ f)(x) = f(f(x)) = f\left(\frac{x}{x-1}\right) = \frac{\frac{x}{x-1}}{\frac{\frac{x}{x-1}-1}{\frac{x}{x-1}-1}} = \frac{\frac{x}{x-1}}{\frac{x - (x-1)}{x-1}} = \frac{x}{x-x+1} = \frac{x}{1} = x$$

$$\left. \begin{array}{l} \text{Domain } f(x) = \mathbb{R} - \{1\} \\ \text{Domain } x = \mathbb{R} \end{array} \right\} \text{Domain } (f \circ f)(x) = \mathbb{R} - \{1\}$$

$$\text{Ex: } f(x) = x^2 + 1 \quad ; \quad g(x) = \sqrt{x-1}$$

$$a) (f \circ g)(x) = f(\sqrt{x-1}) = (\sqrt{x-1})^2 + 1 = x - 1 + 1 = \underline{\underline{x}}$$

$$\left. \begin{array}{l} \text{Domain } g(x) \quad \left. \begin{array}{l} x-1 \geq 0 \\ x \geq 1 \end{array} \right\} [1, \infty) \\ \text{Domain } x = \mathbb{R} \end{array} \right\} (f \circ g)(x) = [1, \infty)$$

Ex: $f(x) = \frac{2x-1}{x-2}$ $g(x) = \frac{x+4}{2x-5}$

a) $(f \circ g)(x) = f(g(x)) = f\left(\frac{x+4}{2x-5}\right) = \frac{2 \frac{x+4}{2x-5} - 1}{\frac{x+4}{2x-5} - 2} \cdot \frac{(2x-5)}{(2x-5)}$

$$= \frac{2(x+4) - (2x-5)}{x+4 - 2(2x-5)} = \frac{2x+8-2x+5}{x+4-4x+10}$$

$$= \frac{13}{-3x+14}$$

Domain $g(x) = \mathbb{R} - \left\{ \frac{5}{2} \right\}$

$2x-5=0$
 $x = \frac{5}{2}$

Domain $\frac{13}{-3x+14} = \mathbb{R} - \left\{ \frac{14}{3} \right\}$

Domain $(f \circ g)(x)$
 $\mathbb{R} - \left\{ \frac{5}{2}, \frac{14}{3} \right\}$

$$-3x+14=0$$

$$\frac{-3x}{-3} = \frac{-14}{-3}$$

DECOMPOSITION OF A COMPOSITE FUNCTION

$$f \circ g = H \Rightarrow H(x) \begin{cases} f(x) \\ g(x) \end{cases}$$

$$\text{Ex: } H(x) = \sqrt{x^2 + 1} \begin{cases} f(x) = \sqrt{x} = \sqrt{x+1} \\ g(x) = x^2 + 1 = x^2 \end{cases}$$