5.1 POLYNOMIAL FUNCTIONS AND MODELS

Def: $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0$

is a polynomial function, where $a_n, a_{n-1}, a_{n-2}, \dots, a_2, a_1, a_0$ are real numbers, n is a positive integer, $a_n \neq 0$

$$f(x) = \underbrace{a_n}_{\text{leading coefficient}} x^{n} + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + \underbrace{a_0}_{\text{constant}}$$

$n \rightarrow$ degree of the polynomial

Ex: $2x^6 + \frac{1}{4}x^5 - 3x^4 + \sqrt{5}\,x^3 - 1x^2 + 0.33x + \pi$

Ex: $3x+2$
- degree 1
- l. coeff. 3
- constant 2.

$x^7 - 8x^4 + 7x^3 - 2x + 3$
- degree 7
- l. coeff 1.
- constant 3

8
- degree 0
- l. coeff. 8
- (constant) 8

Ex:

$f(x) = 4x^5 - 2x^4 + 3x^3 + 9$ - polynomial.

$f(x) = 3x^4 + \underline{2x^{\frac{1}{2}}} - 3x + 4$ - not polynomial.

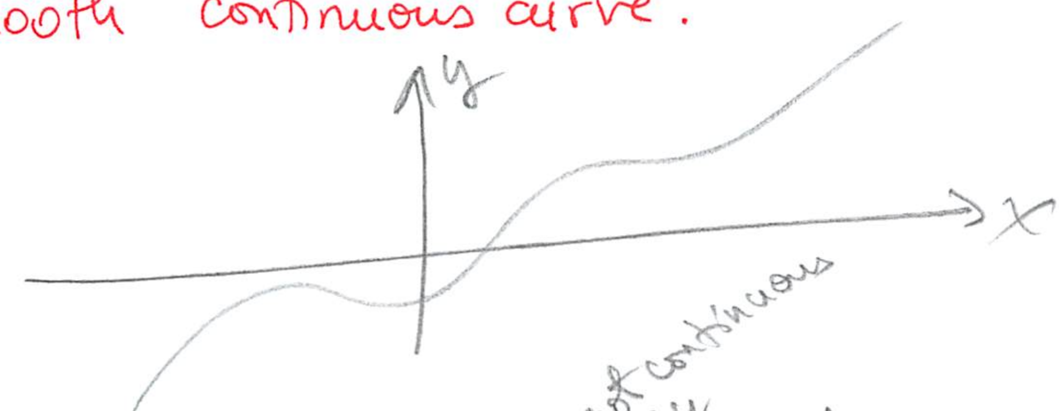
$$x^{\frac{1}{2}} = \sqrt{x}$$

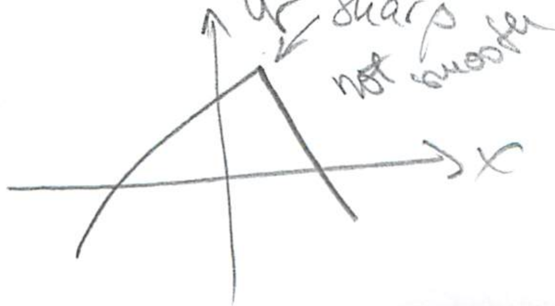
$f(x) = 3x^2 - x^{-1} + x - 3$ - not polynomial.

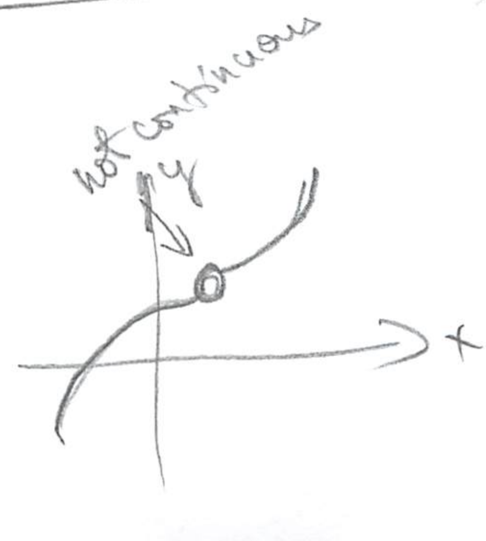
$$x^{-1} = \frac{1}{x}$$

$f(x) = \dfrac{x^3 - 2x^2 + x - 1}{4x - 2}$ not polynomial

The graph of a polynomial function is a smooth continuous curve.







POWER FUNCTION

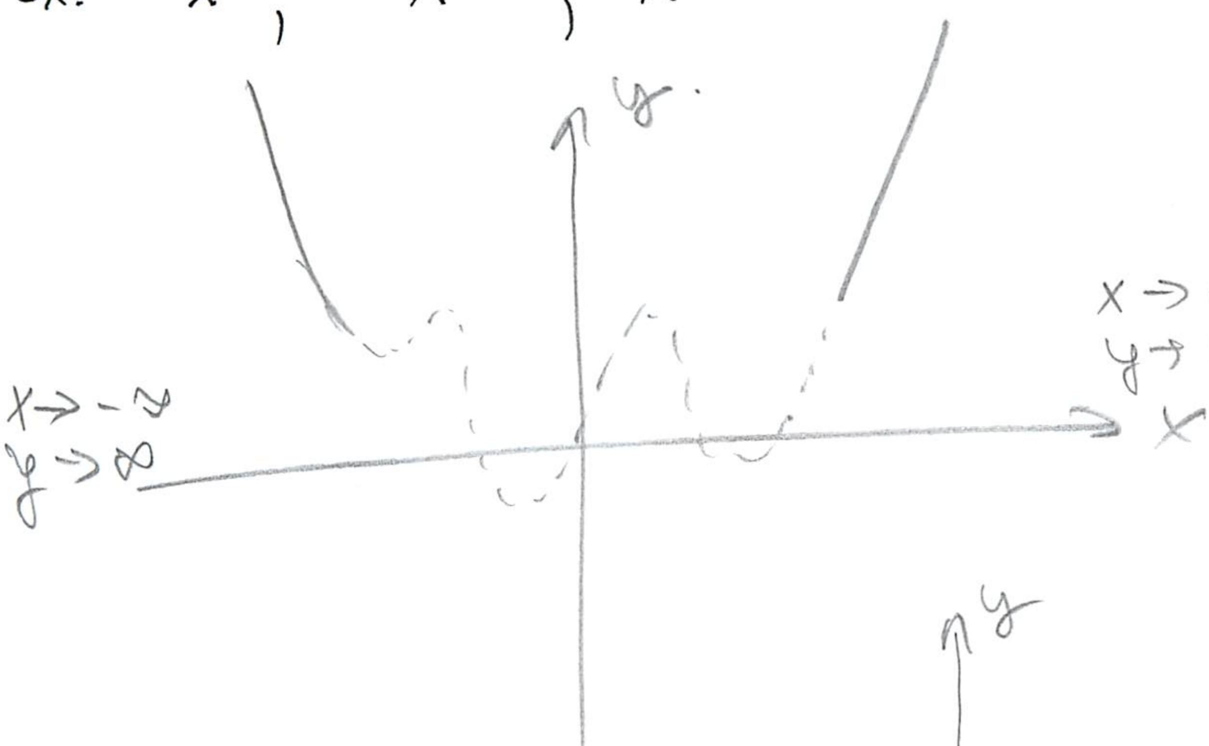
$$f(x) = a_n x^n \qquad a_n \neq 0$$

LONG-RUN BEHAVIOR (END BEHAVIOR)

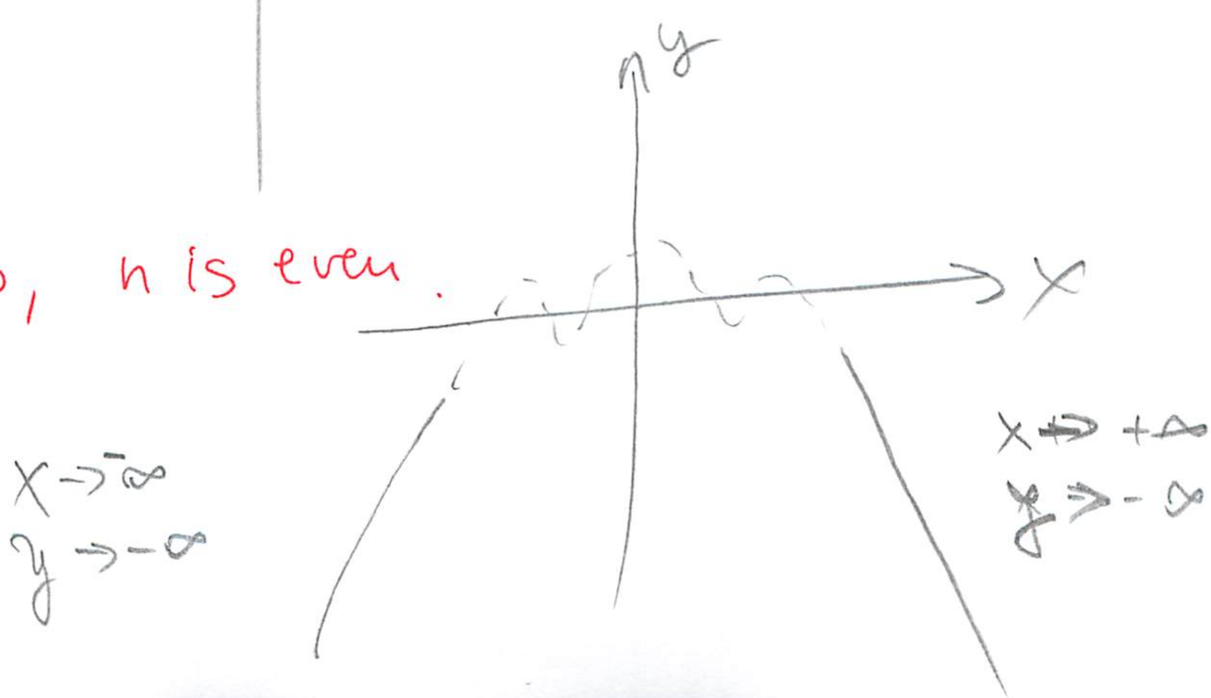
Case I

a) $a_n > 0$, n is even

Ex: x^2, x^{50}, x^{100}



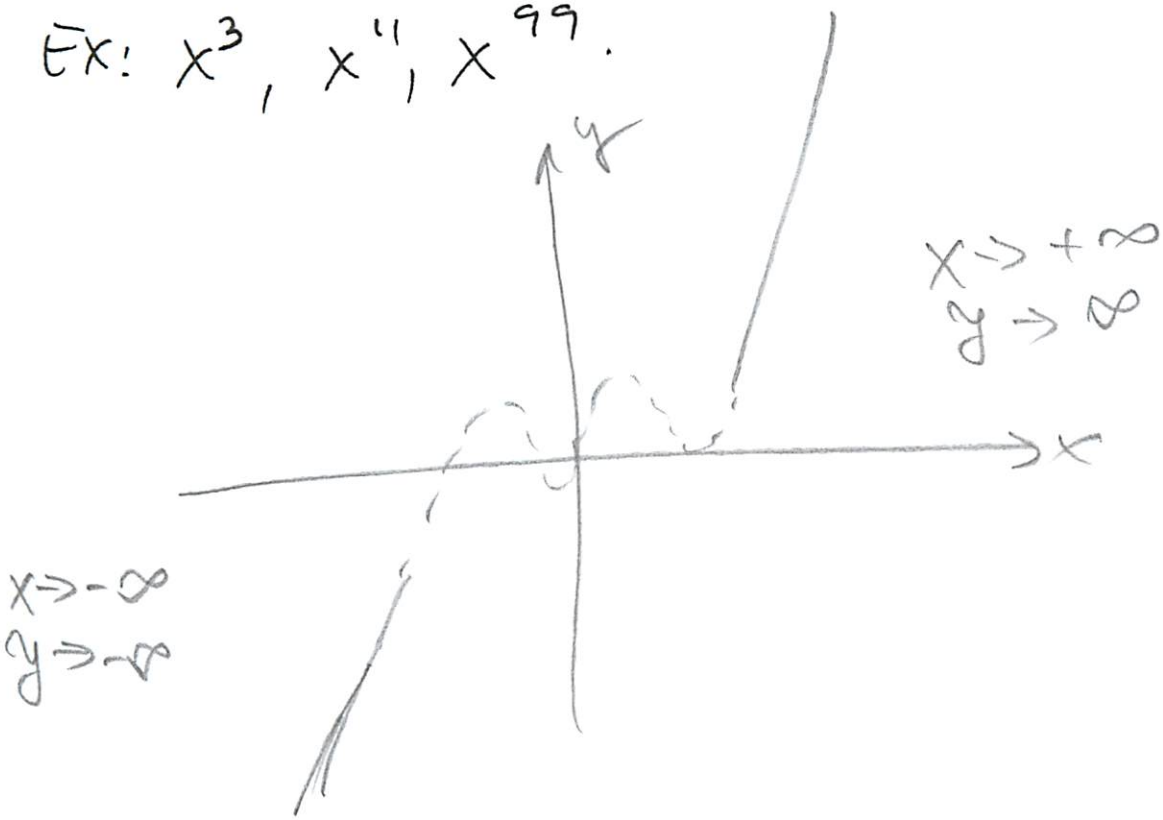
b) $a_n < 0$, n is even.



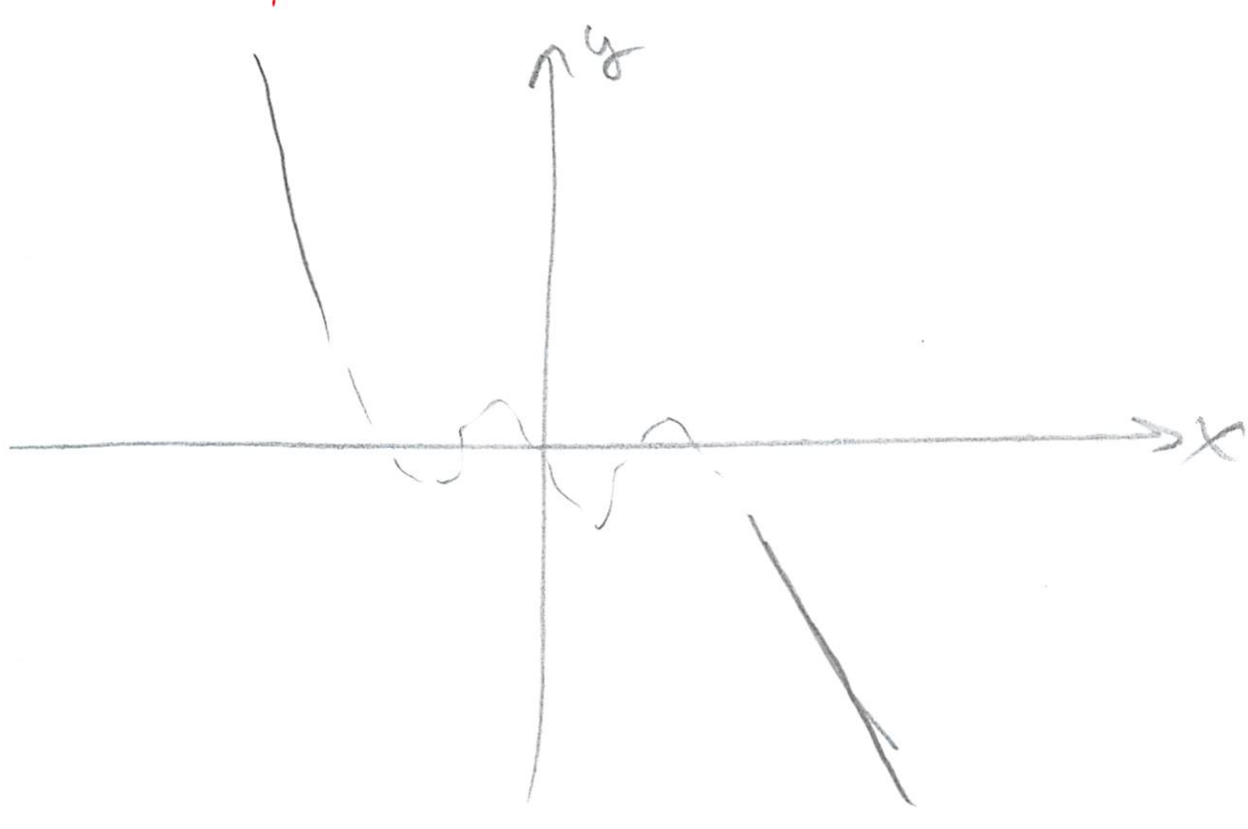
Case II

a) $a_n > 0$, n is odd.

Ex: x^3, x^{11}, x^{99}.



b) $a_n < 0$, n is odd.



A function of n degree has at most $\boxed{n-1}$ turning points

Ex.: $f(x) = x^2 - 3x + 1$ 1 turning point.

$f(x) = x^3$ 2 turning points

$f(x) = x^7 + 6x^3 - 2x + 1$ 6 turning points

SHORT-RUN BEHAVIOUR (INTERCEPTS)

1. y-intercept $\Rightarrow x = 0$

$f(0) \Rightarrow y$ $(0, y)$

2. x-intercept $\Rightarrow y = 0$ (ZEROS OF POLYNOMIAL FUNCTION)

Theorem: if c is a real number and c is a zero of a function $f(x)$ then

a) $f(c) = 0$

b) $(x - c)$ is a factor of $f(x)$

c) $x = c$ is an x-intercept $(c, 0)$

d) c is the solution of the equation $f(x) = 0$

Ex: $f(x) = x^2 + 7x + 12$ $\quad\quad c = -4$

$$f(-4) = (-4)^2 + 7(-4) + 12$$
$$= 16 - 28 + 12$$
$$= 28 - 28 = 0$$

$$x^2 + 7x + 12 = 0$$
$$(x+4)(x+3) = 0$$
$$[x - (-4)] - \text{factor}$$

$x + 4 = 0 \Rightarrow x = -4 \quad (-4, 0)$

$x + 3 = 0 \Rightarrow x = -3 \quad (-3, 0)$

Ex: Find the polynomial function $f(x)$ whose zeros and degree are given

zeros: $-1, 1, 4$ $\quad\quad$ degree 3

$$f(x) = (x+1)(x-1)(x-4)$$
$$= (x^2 - 1)(x - 4)$$
$$= x^3 - 4x^2 - x + 4$$

Ex: $f(x) = x^3 + 2x^2 + x$

$= x(x^2 + 2x + 1)$

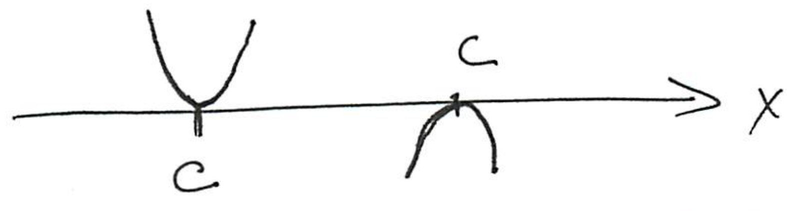
$= x(x+1)^2$

$x = 0$

$x + 1 = 0 \Rightarrow x = -1$ double

Theorem: if $(x-c)^k$ is a factor of $f(x)$ then c is a zero of multiplicity k

1. if k is even then the graph of $f(x)$ will touch the x-axis at $x = c$



2. if k is odd then the graph of $f(x)$ will cross the x-axis at $x = c$

Ex: 78/339

$f(x) = x^2 (x-3)(x+4)$

STEP 1 degree 4 $\Rightarrow$ looks like parabola (x^2)

STEP 2 y-int. $f(0) = 0^2 (0-3)(0+4)$

$= 0 \cdot (-3)(4) = 0 \qquad (0,0)$

x-int. $x^2 (x-3)(x+4) = 0$

$x^2 = 0 \quad x = 0 \qquad (0,0)$

$x-3 = 0 \Rightarrow x = 3 \qquad (3,0)$

$x+4 = 0 \Rightarrow x = -4. \qquad (-4,0)$

STEP 3 $x=0$ multiplicity 2 $\Rightarrow$ touches the x-axis

$x=3$ multiplicity 1 $\Rightarrow$ crosses

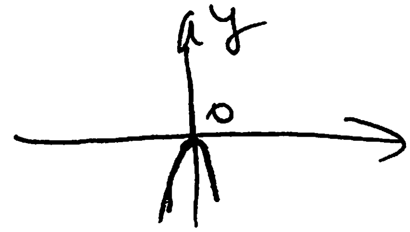
$x=-4$ multiplicity 1 $\Rightarrow$ crosses

STEP 4 $4-1 = 3$ turning points

STEP 5

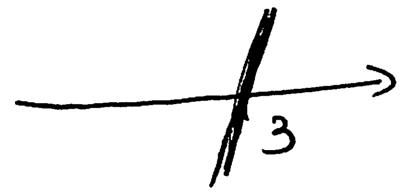
$x=0$

$x^2(0-3)(0+4) = -12x^2$



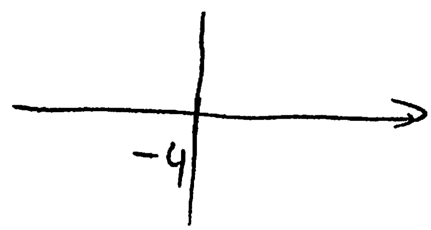
$x=3$

$3^2(x-3)(3+4) = 9\cdot 7(x-3) = 63(x-3) = 63x - \ldots$



$x=-4.$

$(-4)^2(-4-3)(x+4) = -112(x+4) = -112x - \ldots$



STEP 6