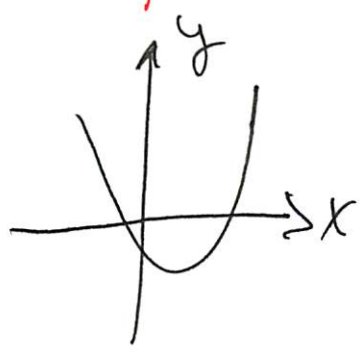
QUADRATIC FUNCTIONS AND THEIR PROPERTIES

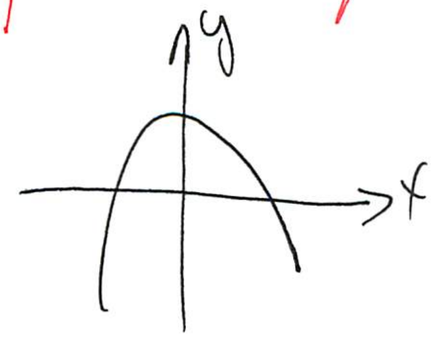
Def: $f(x) = ax^2 + bx + c$ is a quadratic function where $a, b, c \in \mathbb{R}$, $a \neq 0$

The graph of a quadratic function is a parabola.

1. if $a > 0$ parabola opens upward



2. if $a < 0$ parabola opens downward.



Vertex

$$\boxed{\begin{array}{l} V\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) \\ V\left(-\frac{b}{2a}, -\frac{b^2-4ac}{4a}\right) \end{array}}$$

Standard for of a quadratic equation

$$f(x) = a(x-h)^2 + k$$

$$V: (h, k)$$

Ex: $f(x) = x^2 - 6x - 1$

$a = 1$
$b = -6$
$c = -1$

① $-\frac{b}{2a} = -\frac{-6}{2} = \frac{6}{2} = 3$

$f(3) = 3^2 - 6 \cdot 3 - 1 = 9 - 18 - 1 = -10$

$(3, -10)$

② $-\frac{b}{2a} = 3$

$(3, -10)$

$-\frac{36+4}{4} = -\frac{40}{4} = -10$

③ $f(x) = \underbrace{x^2 - 6x + \frac{9}{\quad}}\ - \frac{9}{\quad} - 1$

$a^2 \quad 2ab$

$= (x-3)^2 - 10$

$h \quad k$

V. $(3, -10)$

$f(x) = -2x^2 + 6x + 2$

$= -\overset{a}{2}(x^2 - 3x) + 2$

$= -2\left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right) + 2$

$= -2\left(x^2 - 3x + \frac{9}{4}\right) + (-2)\left(-\frac{9}{\cancel{4}2}\right) + 2$

$= -2\left(x - \frac{3}{2}\right)^2 + \frac{9}{2} + 2$

$= -2\left(x - \frac{3}{2}\right)^2 + \frac{13}{2}$

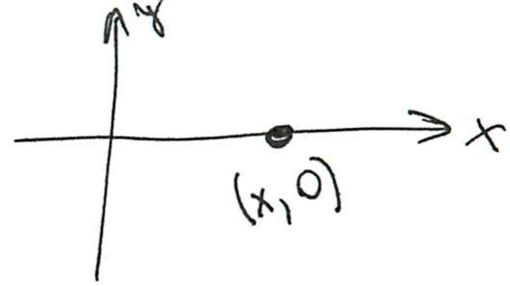
$g = x$

$\cancel{2fg} = 3\cancel{x}$

$2f = 3$

$f = \frac{3}{2}$ $\quad f^2 = \frac{9}{4}$

INTERCEPTS



x-intercept $\Rightarrow y = 0 \quad (f(x) = 0)$

Ex: $f(x) = x^2 + 5x - 6$

$$x^2 + 5x - 6 = 0$$

$$(x+6)(x-1) = 0$$

$x + 6 = 0 \Rightarrow x = -6 \qquad (-6, 0)$

$x - 1 = 0 \Rightarrow x = 1 \qquad (1, 0)$

if $b^2 - 4ac > 0 \Rightarrow$ 2 x-intercepts

if $b^2 - 4ac = 0 \Rightarrow$ 1 x-intercept

$$f(x) = x^2 - 6x + 9$$

$b^2 - 4ac = 36 - 36 = 0$

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0$$

$x = 3 \Rightarrow (3, 0)$ - vertex

if $b^2 - 4ac < 0 \Rightarrow$ no x - intercepts

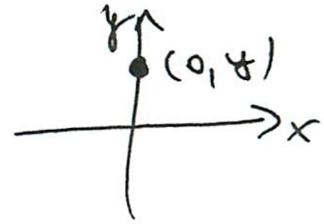
$$f(x) = x^2 - x + 11$$

$$x = \frac{1 \pm \sqrt{1 - 44}}{2} = \frac{1 \pm \sqrt{-43}}{2} \quad {-43} < 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

y-int , $\Rightarrow x = 0$

Evaluate $f(0)$



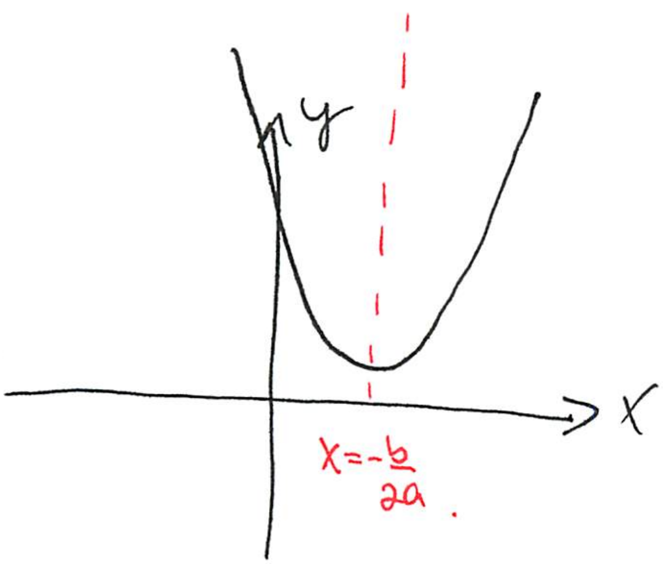
Ex: $f(x) = x^2 - x + 11$ $\quad (0, 11)$

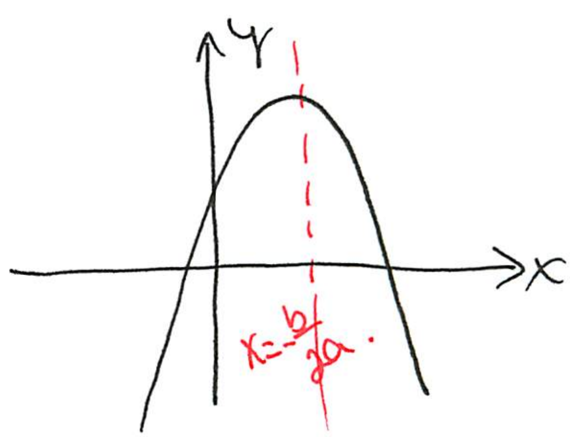
$f(0) = 11$

AX OF SYMMETRY

is a vertical line $\boxed{x = -\frac{b}{2a}}$

x coordinate of the vertex





THE GRAPH

Ex: $f(x) = x^2 + 4x + 7$

x-int: $x^2 + 4x + 7 = 0$

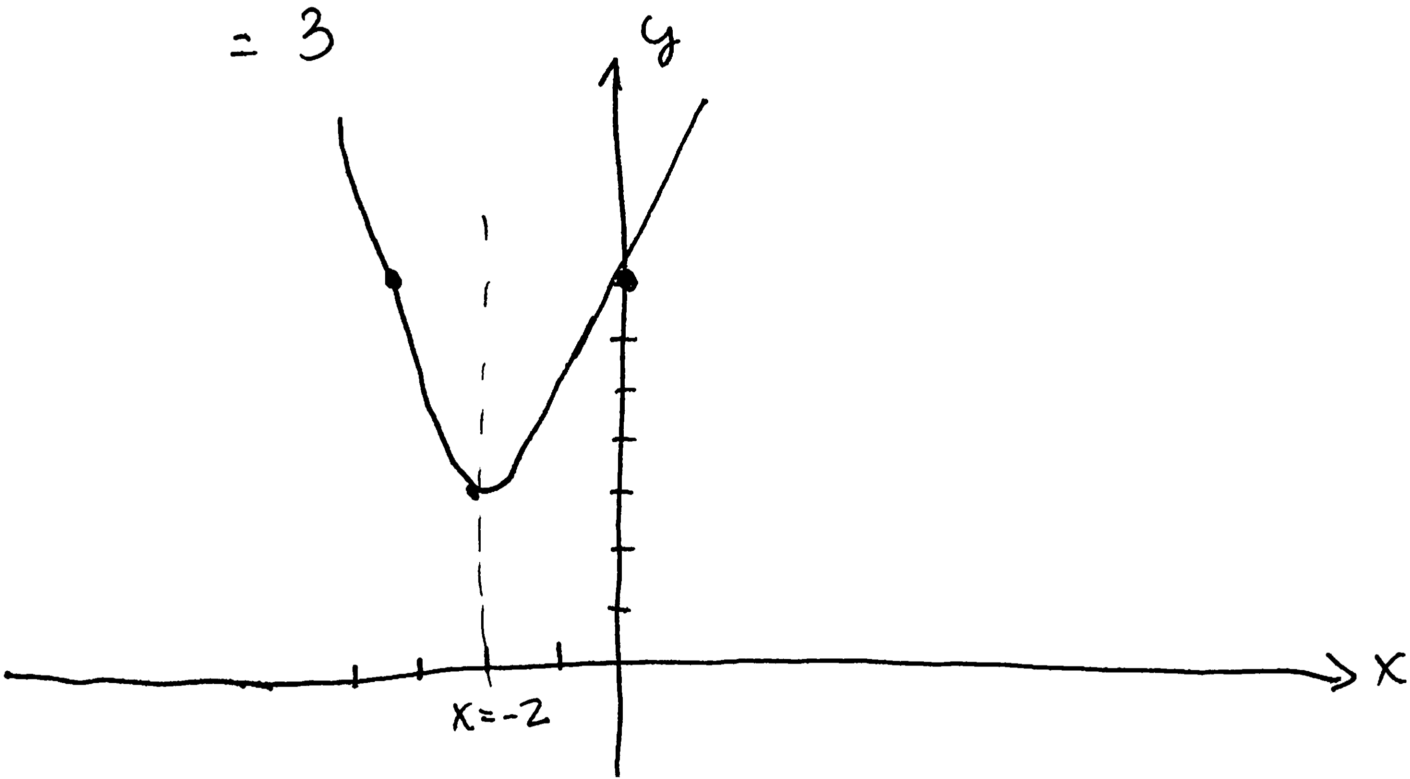
$$x = \frac{-4 \pm \sqrt{16-24}}{2} = \frac{-4 \pm \sqrt{-8}}{2}$$ no x-int.

y-int $f(0) = 7$ $(0, 7)$

Vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = (-2, 3)$

$$-\frac{b}{2a} = -\frac{4}{2} = -2$$

$$f(-2) = (-2)^2 + 4(-2) + 7$$
$$= 3$$

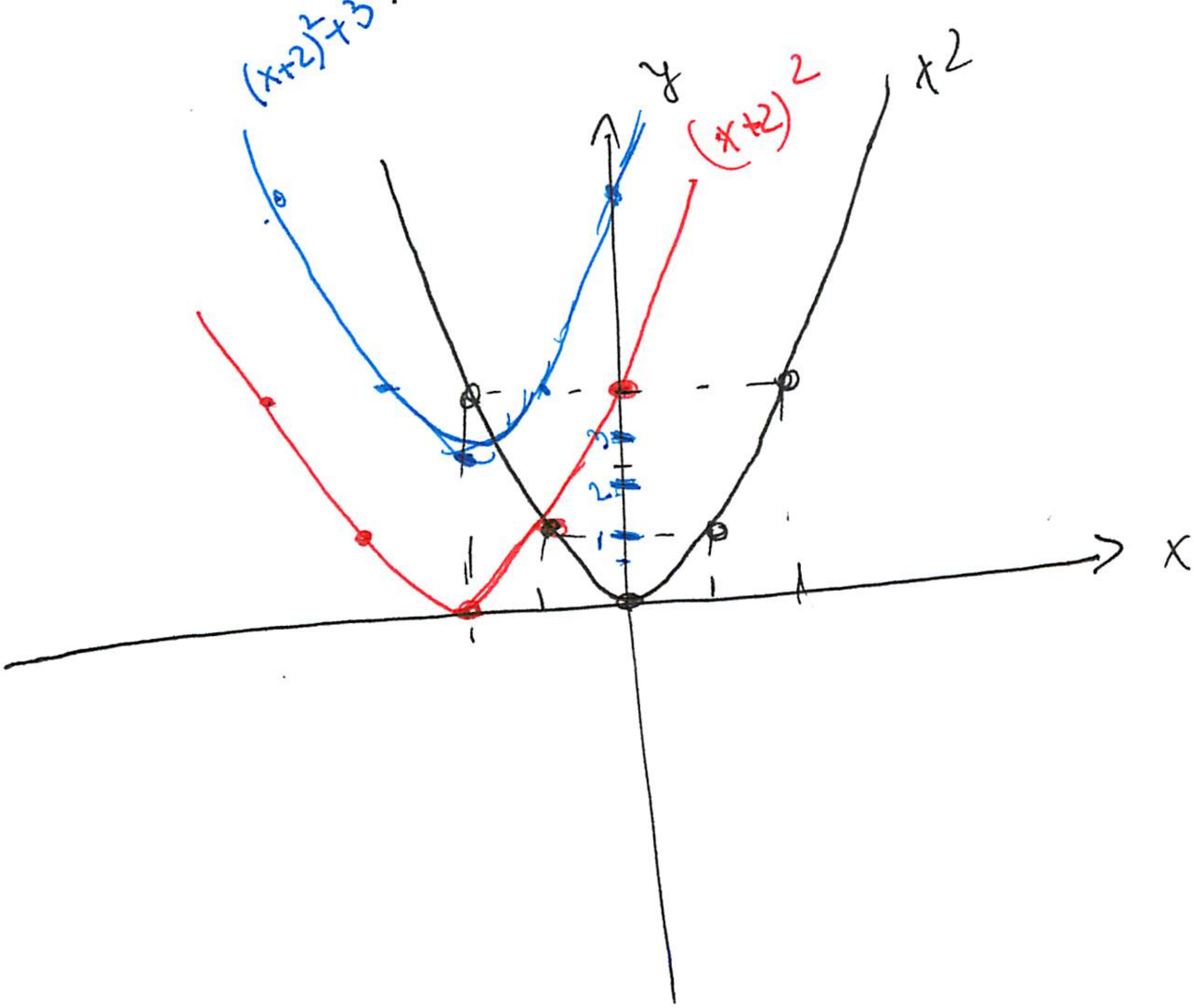


- us transformation

$2fg = 4x$
$f = x$
$2g = 4$
$g = 2$

$$f(x) = \underbrace{x^2 + 4x} + 7$$
$$= \underbrace{x^2 + 4x + \underline{4}} - \underline{4} + 7$$
$$= (x+2)^2 + 3$$

P.F x^2
H.S 2units left.
V.S 3units up.



Ex 48 $\quad v\,(2,1)$

y-int $(0,5)$

$$f(x) = a(x-h)^2 + k$$

$$f(x) = a(x-2)^2 + 1$$

$$5 = a(0-2)^2 + 1$$

$$5 = 4a + 1$$

$$-1 \qquad\qquad -1$$

$$4 = 4a$$

$$a = 1.$$

$$f(x) = 1(x-2)^2 + 1$$

$$= x^2 - 4x + 4 + 1$$

$$= x^2 - 4x + 5$$

Ex: $f(x) = \underbrace{2x^2 + 7x} - 12$

$$= 2\left(x^2 + \frac{7}{2}x + \frac{49}{16} - \frac{49}{16}\right) - 12$$

$$= 2\left(x^2 + \frac{7}{2}x + \frac{49}{16}\right) - \frac{49}{8} - \frac{12}{1}$$

$$= 2\left(x + \frac{7}{4}\right)^2 - \frac{49}{8} - \frac{96}{8}$$

$$= 2\left(x + \frac{7}{4}\right)^2 - \frac{145}{8}$$