

RATIONAL EXPONENTS

AND

RADICALS

$x^m \cdot x^n = x^{m+n}$	$x^0 = 1$
$\frac{x^m}{x^n} = x^{m-n}$	$x^{-n} = \frac{1}{x^n}$
$(x^m)^n = x^{m \cdot n}$	$x^1 = x$

MORE : $(a^{\frac{1}{n}})^m = a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}}$

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$\sqrt[n]{a^m} = a^{m/n}$$

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, \quad b \neq 0$$

NOTE: m, n ARE ~~INTEGERS~~ NATURAL NUMBERS

$b \neq 0$

Ex1 a) $x^7 \cdot x^9 = x^{7+9} = x^{16}$

b) $\frac{x^7}{x^5} = x^{7-5} = x^2$

c) $(x^2)^4 = x^{2 \cdot 4} = x^8$

d) $x^{-3} = \frac{1}{x^3}$

Ex2 $\left(3^{\frac{1}{5}}\right)^4 = 3^{\frac{4}{5}} = \left(3^4\right)^{\frac{1}{5}}$

Ex3 $\sqrt[3]{5} = 5^{\frac{1}{3}}$

$$\sqrt[6]{10} = 10^{\frac{1}{6}}$$

$$\sqrt{7} = 7^{\frac{1}{2}}$$

Ex4 $\sqrt[3]{5^2} = 5^{\frac{2}{3}}$

$$\sqrt[4]{7^9} = 7^{\frac{9}{4}}$$

Ex5 $\sqrt[3]{4} \cdot \sqrt[3]{5} = \sqrt[3]{4 \cdot 5} = \sqrt[3]{20}$

$$\sqrt{3} \cdot \sqrt{7} = \sqrt{3 \cdot 7} = \sqrt{21}$$

$\sqrt[3]{4} \cdot \sqrt{3}$ (DIFF RADICALS) \rightarrow NOT POSSIBLE TO APPLY FORMULA

Ex 6

$$\sqrt[3]{4} \cdot \sqrt[3]{5}$$

$$= 4^{1/3} \cdot 5^{1/3}$$

$$= (4 \cdot 5)^{1/3}$$

$$= 20^{1/3}$$

RADICAL EXP.

RATIONAL EXP

Ex 7

$$\sqrt[3]{\frac{2}{7}} = \frac{\sqrt[3]{2}}{\sqrt[3]{7}}$$

$$\sqrt{\frac{5}{11}} = \frac{\sqrt{5}}{\sqrt{11}}$$

SAME RADICAL

PERFECT POWERS (SQUARES, CUBIC, ETC.)

$$25 \rightarrow \sqrt{25} = 5$$

$$\sqrt{25} = \sqrt{5^2} = 5^{2/2} = 5^1 = 5$$

$$64 \rightarrow \sqrt[3]{64} = \sqrt[3]{4^3} = 4^{3/3} = 4^1 = 4$$

$$\rightarrow \sqrt[6]{64} = \sqrt[6]{2^6} = 2^{6/6} = 2^1 = 2$$

Remark: $\sqrt[3]{64} = \sqrt[3]{2^6} = 2^{6/3} = 2^2 = 4$

Ex 8

$$27^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^{\cancel{3} \cdot \frac{2}{\cancel{3}}} = 3^2 = 9$$

$$32^{-\frac{1}{5}} = (2^5)^{-\frac{1}{5}} = 2^{\cancel{5} \cdot \frac{-1}{\cancel{5}}} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

$$64^{\frac{2}{3}} = (4^3)^{\frac{2}{3}} = 4^{\cancel{3} \cdot \frac{2}{\cancel{3}}} = 4^2 = 16$$

$$16^{-\frac{3}{4}} = (2^4)^{-\frac{3}{4}} = 2^{\cancel{4} \cdot \frac{-3}{\cancel{4}}} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

Ex 9

$$d^{\frac{1}{2}} \cdot d^{\frac{2}{3}} \cdot d^{-\frac{1}{4}} = d^{\frac{1}{2} + \frac{2}{3} - \frac{1}{4}}$$

$$= d^{\frac{6}{12} + \frac{8}{12} - \frac{3}{12}}$$

$$= d^{\frac{6+8-3}{12}}$$

$$= d^{\frac{11}{12}}$$

Ex 10

$$\frac{x^{\frac{1}{2}} \cdot y^{-\frac{5}{4}}}{x^{-\frac{4}{3}} \cdot y^{\frac{1}{3}}}$$

$$= x^{\frac{1}{2} - (-\frac{4}{3})} \cdot y^{-\frac{5}{4} - \frac{1}{3}}$$

$$= x^{\frac{1}{2} + \frac{4}{3}} \cdot y^{-\frac{5}{4} - \frac{1}{3}}$$

$$= x^{\frac{2}{6} + \frac{8}{6}} \cdot y^{-\frac{15}{12} - \frac{4}{12}}$$

$$= x^{\frac{10}{6}} \cdot y^{-\frac{19}{12}} = \frac{x^{\frac{5}{3}}}{y^{\frac{19}{12}}}$$

$$\underline{\underline{\text{Ex 11}}} \quad x^{\frac{1}{4}} \cdot x^{\frac{3}{4}} = x^{\frac{1}{4} + \frac{3}{4}} = x^{\frac{4}{4}} = x^1 = x$$

$$\begin{aligned} \underline{\underline{\text{Ex 12}}} \quad & (-64 x^6 y^{-\frac{3}{2}})^{\frac{4}{3}} \\ &= (-64)^{\frac{4}{3}} \cdot (x^6)^{\frac{4}{3}} \cdot (y^{-\frac{3}{2}})^{\frac{4}{3}} \\ &= (-4)^{\cancel{2} \cdot \frac{4}{2}} \cdot x^{\cancel{6} \cdot \frac{4}{3}} \cdot y^{-\cancel{3} \cdot \frac{4}{2}} \\ &= (-4)^4 \cdot x^8 \cdot y^{-2} = 256 x^8 y^{-2} = \frac{256 x^8}{y^2} \end{aligned}$$

$$\begin{aligned} \underline{\underline{\text{Ex 13}}} \quad & (x^{\frac{3}{4}} \cdot y^{\frac{1}{2}} \cdot z^{-\frac{2}{3}})^{-\frac{4}{3}} \\ &= x^{\frac{3}{4} \cdot -\frac{4}{3}} \cdot y^{\frac{1}{2} \cdot -\frac{4}{3}} \cdot z^{-\frac{2}{3} \cdot -\frac{4}{3}} \\ &= x^{-1} \cdot y^{-\frac{2}{3}} \cdot z^{\frac{8}{9}} = \frac{z^{\frac{8}{9}}}{x y^{\frac{2}{3}}} \end{aligned}$$

$$\underline{\underline{\text{Ex 14}}} \quad \left(\frac{8 a^3 b^{-4}}{64 a^{-9} b^2} \right)^{\frac{2}{3}} = \left(\frac{1}{8} \cdot a^{3-9} \cdot b^{-4-2} \right)^{\frac{2}{3}}$$

$$= \left(\frac{1}{2^3} \cdot a^{12} \cdot b^{-6} \right)^{2/3}$$

$$= \left(2^{-3} \cdot a^{12} \cdot b^{-6} \right)^{2/3}$$

$$= 2^{-3 \cdot \frac{2}{3}} \cdot a^{12 \cdot \frac{2}{3}} \cdot b^{-6 \cdot \frac{2}{3}}$$

$$= 2^{-2} \cdot a^8 \cdot b^{-4}$$

$$= \frac{a^8}{2^2 b^4} = \frac{a^8}{4b^4}$$

Ex 15

$$\frac{(3x^{\frac{3}{4}} \cdot y^{-\frac{1}{4}})^4}{(9x^2y^4)^{\frac{1}{2}}} = \frac{3^4 \cdot x^{\frac{3 \cdot 4}{4}} \cdot y^{-\frac{1 \cdot 4}{4}}}{9^{\frac{1}{2}} \cdot x^{2 \cdot \frac{1}{2}} \cdot y^{4 \cdot \frac{1}{2}}}$$

$$9^{\frac{1}{2}} = (3^2)^{\frac{1}{2}} = 3^{2 \cdot \frac{1}{2}} = 3$$

$$\rightarrow = \frac{3^4 \cdot x^3 \cdot y^{-1}}{3 \cdot x^1 \cdot y^2}$$

$$= 3^{4-1} \cdot x^{3-1} \cdot y^{-1-2}$$

$$= 3^3 \cdot x^2 \cdot y^{-3}$$

$$= \frac{27 \cdot x^2}{y^3}$$

Ex 16

$$a) (3x)^{\frac{2}{3}} \xrightarrow{\text{RADICAL}} \sqrt[3]{(3x)^2} \\ = \sqrt[3]{9x^2}$$

$$b) -2x^{\frac{2}{3}} \xrightarrow{\text{RADICAL}} -2\sqrt[3]{x^2}$$

Ex 17

$$\sqrt[7]{x^5} \xrightarrow{\text{EXPON}} x^{\frac{5}{7}}$$

$$\sqrt[3]{2x} \xrightarrow{\text{EXPON}} (2x)^{\frac{1}{3}}$$

$$2\sqrt[3]{x} \xrightarrow{\text{EXPON}} 2x^{\frac{1}{3}}$$

$$\sqrt[3]{a^2+b^2} \xrightarrow{\text{EXPON}} (a^2+b^2)^{\frac{1}{3}}$$

Ex 18

$$\sqrt{49x^2} = \sqrt{49} \cdot \sqrt{x^2} \\ = 7 \cdot x^{\frac{2}{2}} \\ = \underline{\underline{7 \cdot x}}$$

Remark:

$$\sqrt{x^2} = x$$

$$\sqrt[3]{x^3} = x$$

$$\sqrt[4]{x^4} = x$$

$$\sqrt{x^4} = x^{\frac{4}{2}} = x^2$$

$$\sqrt[3]{x^{12}} = x^{\frac{12}{3}} = x^4$$

Ex 19

$$\begin{aligned} & \sqrt[3]{-125 \cdot a^6 \cdot b^9} \\ &= \sqrt[3]{-125} \cdot \sqrt[3]{a^6} \cdot \sqrt[3]{b^9} \\ &= -5 \cdot a^{6/3} \cdot b^{9/3} \\ &= \underline{\underline{-5a^2b^3}} \end{aligned}$$